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Unsteady conjugated heat transfer in turbulent channel flows with convection from the ambient

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Abstract—A numerical study is carried out to investigate the transient conjugated heat transfer in turbulent channel flows with convection from the ambient. The solutions take wall conduction and heat capacity effects into consideration. Major nondimensional groups identified in this work are the wall-to-fluid conductivity ratio K, the wall-to-fluid thermal diffusivity ratio A, the dimensionless wall thickness β, the Reynolds number Re, the Prandtl number Pr, the outside Nusselt number Nu_o and the dimensionless cooling length L. The influences of wall material, β, Pr, Re and Nu_o on the interfacial heat flux, outer wall temperature and interfacial temperature are examined in detail. Results show that wall conduction plays a significant role in the transient conjugated heat transfer problem. In addition, the time required for heat transfer to reach the steady state condition is longer for systems with a larger β or smaller A, Re and Nu_o.

INTRODUCTION

The growing research on unsteady convective heat transfer is mainly stimulated by the increasing need to procure the precise thermal control of various heat exchange devices encountered in aerospace equipment, nuclear energy systems and chemical processes. Besides, the demands for detailed understanding of the transient heat transfer characteristics in energy related systems during the period of start-up, shutdown, or any off-normal surges in a presumed steady normal operation, possibly resulting from the changes in loading conditions, have significantly increased. In this work, particular attention is given to the investigation of unsteady convective heat transfer in turbulent channel flows with convection from the ambient. The effects of both wall conduction and wall heat capacity are taken into account.

Most heat transfer problems involve an interaction of conduction in a solid wall and convection in an adjacent fluid. The spatial and temporal variations of the thermal conditions along the fluid–wall interface are then not known *a priori*, but what is known is a thermal boundary condition at some other surface of the solid wall. For such cases, it is necessary to solve the energy equations for the fluid and the solid wall under the conditions of continuity in the temperature and heat flux along the fluid–wall interface at every instant of time. Commonly problems of this kind are referred to as the conjugated heat transfer problems.

An integral form of the energy equation was employed by Siegel and Sparrow [1] to examine transient laminar heat transfer in a plate channel subjected to a step change in wall temperature or in wall heat flux. With the assumption of slug flow in a plate channel or a circular pipe, Siegel [2] obtained the solution of unsteady forced convective heat transfer resulting from sudden application of a step-function uniform wall heat flux. Perlmutter and Siegel [3, 4] analyzed the unsteady laminar flow heat transfer in fully-developed flow regime between parallel plates with step changes in both pumping pressure and wall temperature or wall heat flux. Kakac and Yener [5] obtained formal solutions of the energy equation of transient forced convection for the timewise variation of inlet temperature for fully developed turbulent flow between two parallel plates. Lin and Shih [6] used an unsteady local similarity method which is valid for initial small time interval to solve the unsteady laminar convective heat transfer in duct flows. A finite difference solution was performed by Chen *et al.* [7] to investigate unsteady laminar forced convection heat transfer in the thermal entrance region of a circular pipe. Lin *et al.* [8] presented the solutions for the transient laminar forced convection heat transfer in a pipe with convection from the ambient. A second-order finite-difference scheme was proposed by Cotta *et al.* [9] to solve the transient forced convection in laminar channel flow. Analytic solutions were developed by Kim and Ozisik [10] for unsteady laminar forced convection inside circular tubes and parallel plate channels resulting from a step variation in the wall heat flux. In ref. [10], the generalized integral transform technique and the classical Laplace transformation are used to develop a simple lowest order solution as well as higher order solutions. Somasundaram *et al.* [11] presented the numerical solution for unsteady turbulent heat transfer in plate channels subjected to a step change in wall heat flux or wall temperature.

NOMENCLATURE

A	wall-to-fluid thermal diffusivity ratio, α_w/α_f
b	half channel width [m]
C_1, C_2, C_μ	constants appearing in turbulent k–ε model
f_2, f_μ	functions appearing in turbulent k–ε equations
h_o	convective heat transfer coefficient outside wall [W m^{-2} K^{-1}]
K	wall-to-fluid conductivity ratio, λ_w/λ_f
k	turbulent kinetic energy [m^2 s^{-2}]
L	dimensionless cooling length, l/b
l	cooling length [m]
Nu_o	outside Nusselt number, $h_o b/\lambda_f$
Q_{wi}	dimensionless interfacial heat flux
Pr	Prandtl number, ν_f/α_f
Pr_t	turbulent Prandtl number
Re	Reynolds number, $4bu_e/\nu_f$
R_t	turbulent Reynolds number, $k^2/(\nu_f\varepsilon)$
T_e	inlet or initial temperature [K]
T_f	fluid temperature [K]
T_o	ambient temperature [K]
T_w	wall temperature [K]
t	time [s]
u_e	inlet fluid axial velocity [m s^{-1}]
u_*	shear stress velocity, $(\tau_w/\rho)^{1/2}$
U	dimensionless axial velocity, u/u_e
X	dimensionless axial coordinate, x/b
x	axial coordinate [m]
Y	dimensionless transverse coordinate, y/b
y	transverse coordinate [m]
y^+	dimensionless wall coordinate, yu_*/ν_f.

Greek symbols

α_f	thermal diffusivity of fluid [m^2 s^{-1}]
α_w	thermal diffusivity of wall [m^2 s^{-1}]
β	dimensionless wall thickness, δ/b
δ	wall thickness [m]
ε	the rate of dissipation of turbulent kinetic energy [m^2 s^{-3}]
ε^+	dimensionless turbulent eddy viscosity, ν_t/ν_f
λ_f	thermal conductivity of fluid
λ_w	thermal conductivity of channel wall
θ_f	dimensionless fluid temperature, $(T_f-T_o)/(T_e-T_o)$
θ_b	dimensionless bulk temperature
θ_w	dimensionless wall temperature, $(T_w-T_o)/(T_e-T_o)$
θ_{wi}	dimensionless interfacial fluid–wall temperature
θ_{wo}	dimensionless outer wall temperature
ν_f	kinematic viscosity of fluid
ν_t	turbulent eddy viscosity
ρ_f	fluid density [kg m^{-3}]
ρ_w	wall density [kg m^{-3}]
σ_k, σ_ε	constants in turbulent k–ε equations
τ	dimensionless time, $t/(b^2/\alpha_f)$.

Subscripts

b	bulk quantity
e	initial value at the entrance of the channel
f	fluid
o	condition at outer surface
w	wall
wi	fluid–wall interface
wo	outer surface of the channel wall.

Recently, Kim and Ozisik [12] studied analytically the unsteady turbulent forced convection heat transfer in the parallel-plate channel with a periodic variation of inlet temperature.

In the studies [1–12] just mentioned the transient energy equation for the fluid alone was solved with the thermal boundary condition at the fluid–wall interface prescribed as it is at the outer surface of the channel wall. The results of such investigations are only good for heat transfer in flows bounded by extremely thin walls. In practical situations the channel wall is normally finite in thickness, and thereby the thermal resistance associated with the conduction heat transfer in a solid wall and the process of thermal energy stored in the wall during the transient must be included in the analysis.

The influences of heat capacity of a duct wall on the transient forced convection heat transfer in duct flows have been examined by Kawamura [13, 14], Sucec [15–18], Cotta *et al.* [19], Kim and Ozisik [20], Li and Kakac [21], Travelho and Santos [22] and Brown *et al.* [23]. These contributions showed that it is of great importance. Nevertheless, wall conduction still remained untreated. The problems of unsteady laminar forced convection heat transfer in a duct with wall conduction effects were investigated by Krishan [24], Lin and Kuo [25], Olek *et al.* [26], Guedes and Cotta [27], Yan [28] and Schutte *et al.* [29]. Despite the relative importance in engineering applications, the unsteady conjugated heat transfer in the turbulent channel flows has not received much attention. This motivates the present study to examine the transient conjugated heat transfer in turbulent duct flows with convection from the ambient.

ANALYSIS

The physical model chosen for this study is an infinitely long channel ($-\infty < x < \infty$) with half channel width b and wall thickness δ, a schematic diagram of which is shown in Fig. 1. Initially, the system comprising the flowing fluid and the confining channel wall is at a constant and uniform temperature T_e. The flow enters the channel with a uniform velocity u_e and a uniform temperature T_e in the far upstream end of the channel $(x \to -\infty)$. The channel is directly exposed to the ambient with an external heat transfer coefficient h_0 over a finite length $(0 \leqslant x \leqslant l)$ and is externally insulated both upstream $(-\infty < x < 0)$ and downstream $(l < x < \infty)$ of the channel. At time $t = 0$, the ambient temperature is suddenly changed to a new level T_o. Heat is transferred in the channel wall both upstream and downstream of the cooling section by axial wall conduction. Heat is transported to the solid wall by convection heat transfer at the solid–fluid interface. In the fluid domain, the heat is transferred in the direction of the flow by advection and conduction. Attention is focused on the unsteady thermal interactions between the conduction heat transfer in the channel wall and the convective heat transfer in the fluid through their common interface.

Since the fluid is assumed to enter the channel in the far upstream region, the flow can be regarded as hydrodynamically fully developed in the region where heat transfer is significantly present. The physical properties of the fluid and the channel wall are assumed to remain constant. Viscous dissipation and conduction heat transfer along the direction of the fluid flow are also neglected. With these simplifying assumptions, the energy transport processes in the problem treated can be formulated by the following non-dimensional equations:

energy equation for the fluid

$$\partial\theta_f/\partial\tau + (1/4)PrRe \cdot U(Y)\partial\theta_f/\partial X = \partial[(1+Pr\cdot\varepsilon^+/Pr_t)\partial\theta_f/\partial Y]/\partial Y \quad (1)$$

energy equation for the wall

$$\partial\theta_w/\partial\tau = A(\partial^2\theta_w/\partial X^2 + \partial^2\theta_w/\partial Y^2). \quad (2)$$

The initial conditions are

$$\tau = 0 \quad \theta_f = \theta_w = 1 \quad -\infty < X < \infty. \quad (3)$$

The governing equations are subjected to the following boundary conditions:

$$Y = 0, \quad \partial\theta_f/\partial Y = 0, \quad -\infty < X < \infty \quad (4)$$

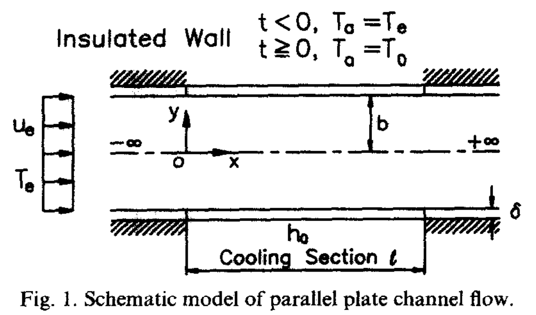

Fig. 1. Schematic model of parallel plate channel flow.

$$Y = 1+\beta, \quad \partial\theta_w/\partial Y = \begin{cases} -Nu_o \cdot \theta_w & 0 < X < L \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$X \to -\infty \quad \theta_f = \theta_w = 1 \quad (6)$$

$$X \to \infty \quad \partial\theta_f/\partial Y = \partial\theta_w/\partial Y = 0 \quad (7)$$

where the outside Nusselt number Nu_o in equation (5) is defined as [8, 17]

$$Nu_o = h_o b/k_w. \quad (8)$$

The conditions of the continuities in temperature and heat flux along the fluid–wall interface are

$$Y = 1 \quad \begin{cases} \theta_f = \theta_w \\ \partial\theta_f/\partial Y = K\partial\theta_w/\partial Y \end{cases} \quad (9)$$

where the dimensionless quantities are defined as

$$\begin{array}{ll} X = x/b & Y = y/b \\ \tau = t/(b^2/\alpha_f) & L = l/b \\ K = \lambda_w/\lambda_f & A = \alpha_w/\alpha_f \\ \beta = \delta/b & Re = 4u_e b/\nu_f \\ \theta = (T-T_o)/(T_e-T_o) & U = u/u_e \\ \varepsilon^+ = \nu_t/\nu_f & Pr = \nu_f/\alpha_f. \end{array} \quad (10)$$

The variables of engineering interest are the interfacial temperature, outer wall temperature and dimensionless interfacial heat flux. These variables are defined as follows

$$\theta_{wi} = \theta_f(X, 1) = \theta_w(X, 1) \quad (11)$$

$$\theta_{wo} = \theta_w(X, 1+\beta) \quad (12)$$

$$Q_{wi} = \partial\theta_f/\partial Y|_{Y=1}. \quad (13)$$

VELOCITY PROFILE AND TURBULENT EDDY VISCOSITY

Prior to solving the problem, the turbulent velocity profile and eddy viscosity for a given Reynolds number are computed in accordance with the k–ε turbulent model. Hence the transport equations for the turbulent kinetic and turbulent energy dissipation must be included in the analysis. To procure more reliable results, a low-Reynolds number k–ε turbulent model is selected to eliminate the usage of wall functions in the computation and thus to permit direct integration of the transport equations to the channel wall. The modified low-Reynolds number k–ε model developed by Myong *et al.* [30, 31] is used in the present study. For the fully-developed channel flow, the governing equations for the velocity fields are:

the axial momentum equation

$$0 = -(1/\rho_f)\,dp/dx + d[(\nu_f+\nu_t)\,du/dy]/dy \quad (14)$$

overall mass conservation at every axial location

$$\int_o^b u\,dy = u_e b \tag{15}$$

the turbulent kinetic energy equation

$$0 = d[(\nu_f+\nu_t/\sigma_k)\,dk/dy]/dy+\nu_t(du/dy)^2-\varepsilon \tag{16}$$

the rate of dissipation of turbulent kinetic energy equation

$$0 = d[(\nu_f+\nu_t/\sigma_\varepsilon)\,d\varepsilon/dy]/dy + C_1(\varepsilon/k)\nu_t(du/dy)^2-C_2 f_2\varepsilon^2/k \tag{17}$$

where

$$\nu_t = C_\mu f_\mu k^2/\varepsilon$$

$$f_2 = \{1-2/9\cdot\exp[-(R_t/6)^2]\}[1-\exp(-y^+/5)]^2$$

$$f_\mu = (1+3.45/\sqrt{R_t})\cdot[1-\exp(-y^+/70)]$$

$$\varepsilon_w = \nu_f(\partial^2 k/\partial y^2)_w \quad R_t = k^2/(\nu_f\varepsilon) \quad y^+ = yu_*/\nu_f. \tag{18}$$

The other empirical constants take the following value [30, 31]

$$\sigma_k = 1.4 \quad \sigma_\varepsilon = 1.3 \quad C_1 = 1.4$$

$$C_2 = 1.8 \quad C_\mu = 0.09 \quad Pr_t = 0.9. \tag{19}$$

SOLUTION METHOD

Because of the complex interactions between the convection heat transfer in the flow and the conduction heat transfer in the channel wall across the fluid–wall interface, the solution of the problem defined by the foregoing equations can be better solved by the numerical finite-difference procedures. Due to the presence of axial heat conduction, equation (2) is a partial differential equation of elliptic type in space. Hence the solution has to be solved simultaneously for all grid points in the channel wall at each time step. For the purpose of numerical stability, a fully implicit formulation in time is adopted. The unsteady energy storage and advection terms are approximated by backward difference and upwind difference, respectively. The axial and transverse diffusion terms are approximated by the central difference. The matching condition imposed at the fluid–wall interface, equation (9), so as to ensure the continuity of heat flux, was recast in backward difference for $\partial\theta_f/\partial Y$ and forward difference for $\partial\theta_w/\partial Y$. Therefore, the solution to the energy equations both in the fluid and the channel wall can be solved simultaneously by the line-by-line method [32]. The finite difference equations have the form

$$a_{j,1}\theta^m_{i,j-1}+a_{j,2}\theta^m_{i,j}+a_{j,3}\theta^m_{i,j+1} = a_{j,4}+a_{j,5}\theta^k_{i-1,j}+a_{j,6}\theta^k_{i+1,j} \tag{20}$$

where $\theta^m_{i,j}$ is the dimensionless temperature at nodal point (i,j) at time m, and i, j, m are the indices in the axial direction, transverse direction and time, respectively. Although the left-hand side of equation (20) is written in tridiagonal form, in fact these finite difference equations are not tridiagonal, as evident from the presence of the last term on the right-hand side of equation (20). But they can be solved by the Thomas algorithm [32], which is a very efficient numerical scheme for tridiagonal matrix equations, with iterations for each time till a certain degree of convergence has been reached.

To obtain desired accuracy, grids are nonuniformly spaced in the axial and transverse directions to account for the uneven variations of θ_f and θ_w. The grid densities are highest near the interface in the transverse direction, and concentration is highest in the neighborhood of $X = 0$ and $X = L$ in the axial direction. To ensure accuracy and reduce computation time, a nonuniform time step is employed. The first time interval $\Delta\tau_1$ is taken to be 0.00001, and every subsequent interval is enlarged by 6% over the previous one, i.e. $\Delta\tau_i = 1.06\Delta\tau_{i-1}$.

During the program tests, solutions for a typical case were obtained using different grid sizes to ensure that the solution is grid-independent. The results from the computation for various grids at several time instants are given in Table 1. It is noted that the differences in the interfacial heat flux Q_{wi} from computations using either $271\times121\times61$ or $181\times81\times41$ grids are always less than 3%. To reduce the cost of computation, the $181\times81\times41$ grid is chosen for the subsequent computations. To further check the adequacy of the numerical scheme for the present study, results for the limiting case of an extremely thin wall were obtained. Excellent agreement between present predictions and those of Somasundaram *et al.* [11] was found. A comparison was also made by comparing the results for the limiting case of laminar conjugated heat transfer with the analytic solutions of Sucec [17] and Kim and Ozisik [20]. The predicted results agreed well with those of Sucec [17] and Kim and Ozisik [20]. Through these program tests, the proposed numerical algorithm is considered to be suitable for the problem.

RESULTS AND DISCUSSION

Inspection of the preceding analysis reveals that the characteristics of transient conjugated heat transfer depend on seven independent parameters, namely, the wall-to-fluid conductivity ratio K, the wall-to-fluid thermal diffusivity A, the dimensionless wall thickness β, the outside Nusselt number Nu_o the Reynolds number Re, Prandtl number Pr and the dimensionless cooling length L. It would be voluminous and impractical to present the results covering the whole range of the above parameters. Instead, to elucidate the characteristics of transient turbulent convective heat transfer, the results will be presented for water ($Pr = 5.0$) or air ($Pr = 0.7$) flowing in a channel with carbon steel wall or aluminum alloy wall of various thickness. The objective here is to present a simple of the transient turbulent convective heat transfer which

Table 1. Comparisons of local interfacial heat flux Q_{wi} for various grid arrangements for $\beta = 0.2$, $Re = 2 \times 10^4$, $Nu_o = 10$ and $Pr = 5.0$ for channel wall of carbon steel

	$I \times J \times M$							
	X							
	$271 \times 121 \times 61$		$181 \times 121 \times 61$		$181 \times 81 \times 41$		$91 \times 41 \times 21$	
τ	2.567	7.433	2.567	7.433	2.567	7.433	2.567	7.433
1.15×10^{-4}	16.94	16.94	16.94	16.94	17.35	17.35	16.62	16.62
2.33×10^{-4}	27.40	27.39	27.40	27.39	27.75	27.74	27.32	27.32
6.37×10^{-4}	35.10	33.91	35.05	33.93	35.18	34.06	35.34	34.41
1.45×10^{-3}	37.30	34.11	37.19	34.19	37.21	34.22	37.38	37.70
3.09×10^{-3}	37.41	34.07	37.31	34.15	37.32	34.18	37.56	34.67
1.97×10^{-2}	37.01	34.04	37.18	34.12	37.19	34.14	37.46	34.65

I, number of grid points in the longitudinal direction.
J, number of grid points in the transverse direction in the fluid side.
M, number of grid points in the transverse direction in the wall side.

takes into account the heat capacity and wall conduction effects. In the following, the cooling length L is fixed to be 10.

Although the local Nusselt number is traditionally considered in the presentation of convective heat transfer results, the local Nusselt number is not a convenient design parameter in the study of unsteady conjugated convective heat transfer [16, 25]. Instead, the interfacial heat flux distributions contain more meaningful information. The unsteady axial distributions of the interfacial heat flux Q_{wi} are shown in Fig. 2 for water ($Pr = 5.0$) flowing in a carbon steel channel ($K = 90$ and $A = 100$) with $\beta = 0.2$, $Re = 2 \times 10^4$ and $Nu_o = 10$ at various instants of time. In the upstream region ($X < 0$), because of the axial wall conduction from the insulated portion to the cooling section, it is of interest to note the presence of some heat transferred from the fluid to wall, even when the Peclet number is so high ($Pe = Re \cdot Pr = 1 \times 10^5$). In the initial transient, only a small amount of energy is dissipated upstream. As time increases, the magnitude of heat flux becomes larger, and the thermal diffusion penetrates further upstream. In the directly cooling region ($0 \leqslant X \leqslant L$), Q_{wi} is rather uniform in the X direction during the early transient ($\tau \leqslant 1.69 \times 10^{-4}$). The uniformity of Q_{wi} indicates that heat transfer in the flow is conduction-dominant at small τ. It is noted that, due to the stronger convection in the turbulent flow, the conduction dominant period is much shorter ($\tau \leqslant 1.69 \times 10^{-4}$), as compared with the results for laminar flow [28]. Later ($\tau \geqslant 1.69 \times 10^{-4}$), convection in the flow becomes important and Q_{wi} is non-uniform in the X direction. Q_{wi} in the cooling section gradually rises from zero to a maximum value with time and approaches the steady-state value. In the neighborhood of the exit end of the cooling section, Q_{wi} shows a drastic change in the flow direction at different moments of time. Q_{wi} is negative for $\tau \leqslant 4.45 \times 10^{-3}$, that is, heat transfer is from the wall to the fluid. This reversal in heat transfer direction is caused by the conjugated nature of the wall conduction and flow convection processes [25, 28].

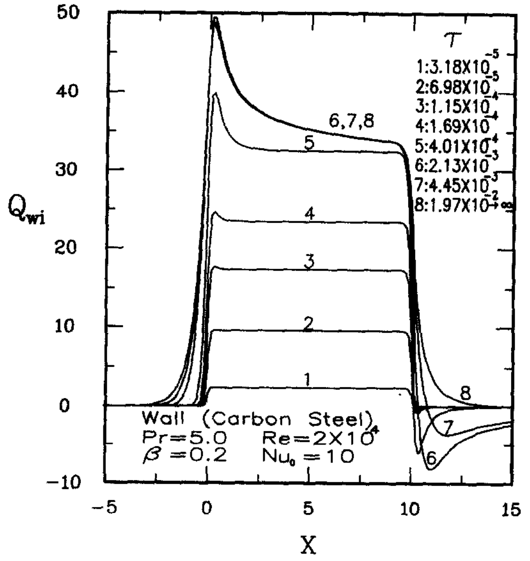

Fig. 2. Transient axial distributions of interfacial heat flux.

Aside from the interfacial heat flux which is important in aspect of thermal interactions between the fluid and the wall, the results for axial distributions of outer wall temperature and interfacial temperature are more descriptive from the viewpoint of understanding the energy transport processes in the system. Hence, to improve our understanding, Fig. 3 gives the results for θ_{wo} and θ_{wi}. As mentioned above, in the early transient period, the heat transfer mode is predominated by heat conduction. This causes flat curves of θ_{wo} and θ_{wi} in the region of direct cooling. But the quantity of heat transported by axial wall conduction from the insulated section is very small so that the values of θ_{wo} and θ_{wi} are small in the insulated portion near the entering end of the cooling section. With time elapsing, the effects of both the convection in the flow and the conduction in the wall increase, resulting in an increase in θ_{wo} and θ_{wi} at the end portions of the

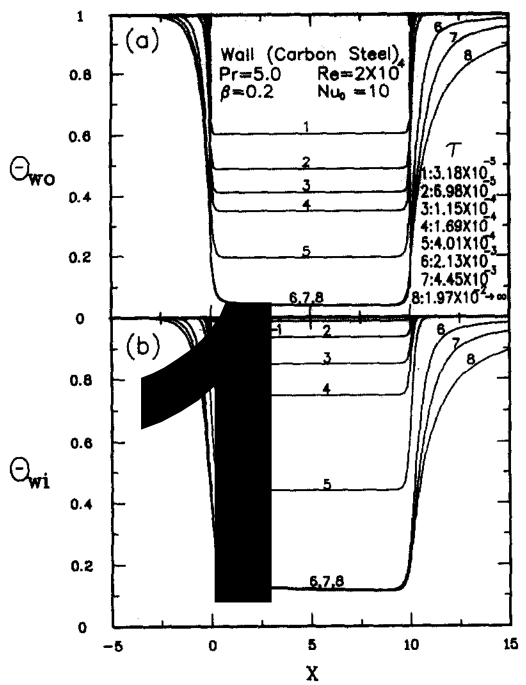

Fig. 3. Unsteady axial distributions of outer and interfacial temperatures.

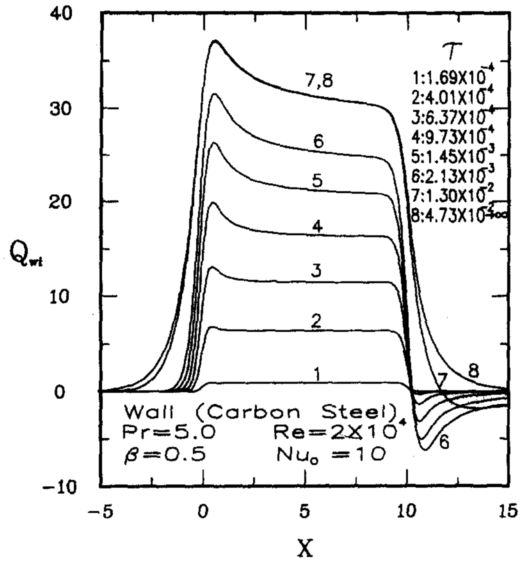

Fig. 4. Effects of dimensionless wall thickness β on the unsteady axial distributions of interfacial heat flux.

cooling section. These two portions get larger with time.

Wall thickness is also an important parameter in conjugated heat transfer problems. Figure 4 presents the predicted Q_{wi} with a thicker wall, $\beta = 0.5$. Comparison of Figs. 2 and 4 shows that when the wall becomes thicker, the heat penetration along the upstream end of the cooling section gets more significant because in this case larger cross section area is provided for axial wall conduction. The energy storage capacity of the wall is larger for a larger β, causing the system to reach the steady state in a longer time period. Additionally, it is found that the value of Q_{wi} at $\tau = 1.69 \times 10^{-4}$, when the heat conduction is still dominant, is larger for the system with a thinner wall ($\beta = 0.2$, Fig. 2). The above outcomes are clearly due to the fact the total thermal resistance and heat capacity of the channel wall are smaller for a thinner wall so that the heat supplied from the wall–fluid interface is easily transported to the outer surface of the channel. Consequently, the presence of the channel wall has a considerable influence on the characteristics of transient heat transfer, and thus the wall effects cannot be neglected for transient convective heat transfer in a turbulent channel flow.

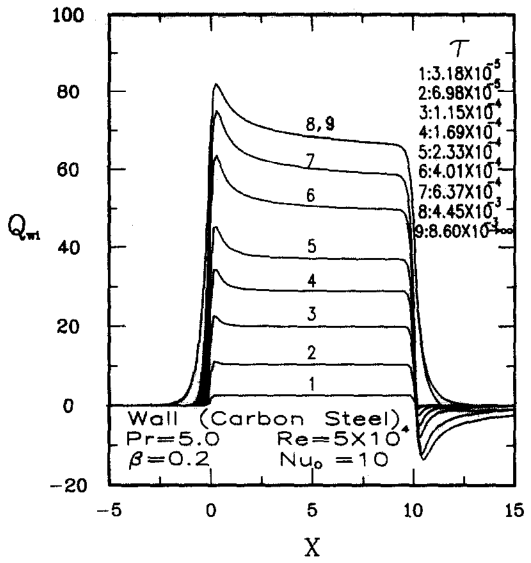

Fig. 5. Effects of Reynolds number Re on the unsteady axial distributions of interfacial heat flux.

Reynolds number Re is another parameter affecting the unsteady conjugated turbulent heat transfer. The unsteady variations of axial interfacial heat flux Q_{wi} are shown in Fig. 5 for $Re = 5 \times 10^4$. Comparing Figs. 2 and 5 indicates that increasing Re from 2×10^4 to 5×10^4 results in a shorter transient period and less significant upstream and downstream energy penetration. This is the direct consequence of the decreasing axial wall conduction in a relative sense when convection in the flow is more effective for a higher Re.

In Fig. 6, the effects of outside Nusselt number Nu_o on the distributions of Q_{wi} are given for water flowing in a carbon steel channel at various instants of time for $Nu_o = 1$. The curves are similar to those for $Nu_o = 10$ (Fig. 2). But there is noticeable difference between them. The time required for the heat transfer to reach the steady-state condition is longer for a smaller Nu_o. This is solely attributed to the decrease in Nu_o which can retard the transverse diffusion process [8].

To examine the influences of the channel wall material on the transient convective heat transfer

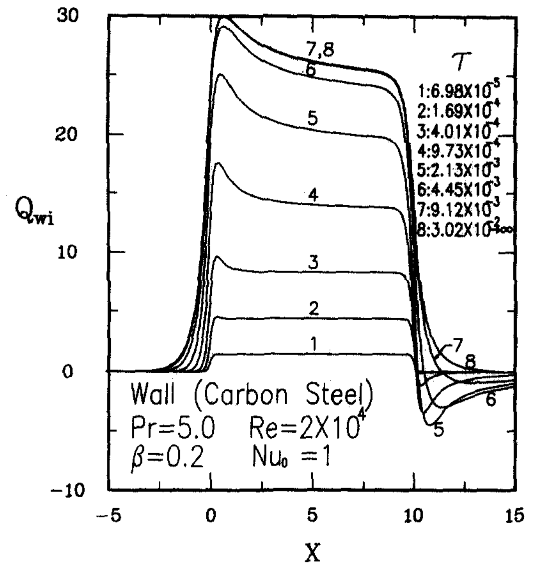

Fig. 6. Effects of outside Nusselt number Nu_o on the unsteady axial variations of interfacial heat flux.

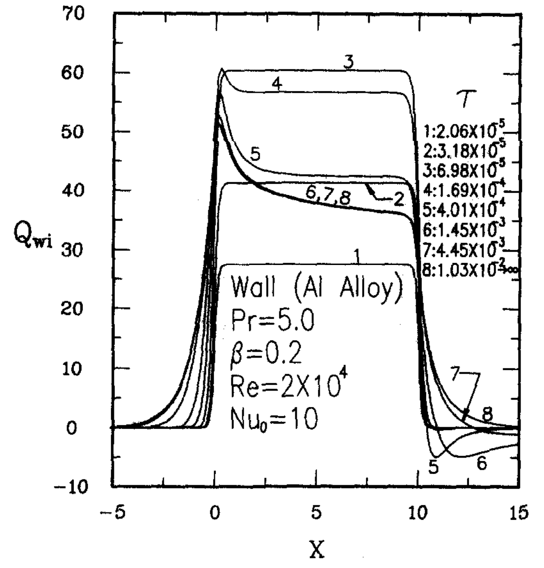

Fig. 7. Effects of wall material on the unsteady axial distributions of interfacial heat flux.

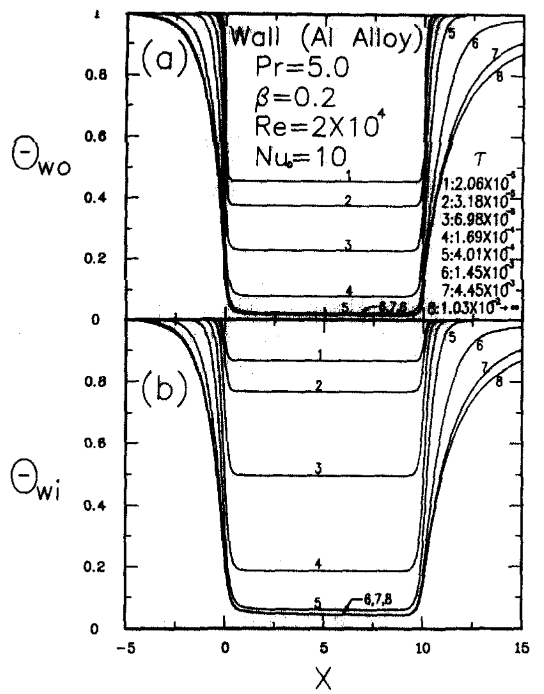

Fig. 8. Effects of wall material on the unsteady axial distributions of outer and interfacial temperatures

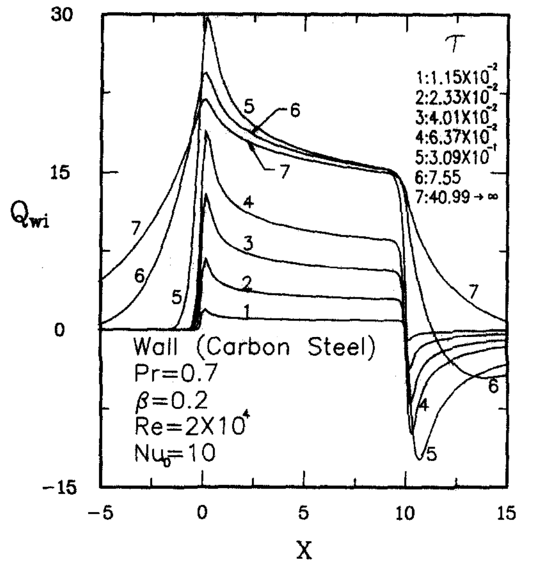

Fig. 9. Effects of Prandtl number Pr on the unsteady axial distributions of interfacial heat flux.

characteristics, the axial distributions of Q_{wi}, θ_{wo} and θ_{wi} are shown in Figs. 7 and 8 at various instants of time for water flowing in a channel with aluminum alloy wall ($K = 250$ and $A = 500$). Particularly noticeable differences between Figs. 2 and 7 or Figs. 3 and 8 are that the heat penetration by the axial conduction in the channel wall becomes much larger for the aluminum alloy channel wall, and the time period to reach steady state is shorter for the aluminum alloy channel wall. These are can be readily understood by recognizing that higher thermal conductivity ratio K and thermal diffusivity ratio A associated with the aluminum alloy wall result in a larger upstream energy penetration and a shorter transient period [28]. Also found in Fig. 7 is that Q_{wi} in the cooling section increases from zero to a maximum value with time. After reaching the maximum, Q_{wi} decreases until it reaches the steady-state value.

The last parameter to be discussed is the Prandtl number Pr. Figure 9 shows the unsteady axial distributions of interfacial heat flux Q_{wi} for air ($Pr = 0.7$) flowing in a channel with carbon steel wall ($K = 1800$ and $A = 0.5$). The smaller Pr results in a larger thermal lag and a larger heat penetration in the upstream and downstream of the cooling section. This is due to the fact that higher K and lower A are experienced for the system with air flowing in a channel.

CONCLUSIONS

A fully implicit finite-difference method is used to numerically study the unsteady conjugated heat transfer in a turbulent channel flow with convection from the ambient. The solutions take wall conduction effects into consideration. The results of interfacial heat flux Q_{wi} outer wall temperature θ_{wo} and interfacial temperature θ_{wi} are presented over wide ranges of the dimensionless governing parameters. The major results can be briefly summarized as follows:

(1) In the cooling section, the interfacial heat flux Q_{Wi} rises from zero to a maximum value with an increase in time. After reaching the maximum, Q_{wi} approaches the steady-state value.

(2) The unsteady variations of Q_{wi} considerably deviate from the corresponding steady state values, especially in the initial transients.

(3) The time required for the heat transfer to reach the steady state condition is longer for the system with a larger β or with smaller A, Re and Nu_o.

(4) The extent of the heat penetration in the upstream region of the cooling section increases with the increase in β or the decrease in Re.

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